# 每日一進

**修行僧人，都能貫徹「一日不作，一日不食」的自律**
**修業儒生，豈能推諉「一日不進，一日不食」的自課**

## 【週六版：大一微積分】

1、Find the limit, if it exists.

(1) $\lim\limits_{x\to 0}\dfrac{\sin\left(x^2\right)}{1-\cos x}$

(2) $\lim\limits_{x\to 0}\dfrac{\left|\tan x\right|}{1-\sqrt{1+2x}}$

(3) $\lim\limits_{x\to\infty}\left(1+\dfrac{2}{x}\right)^{[x]}$

答：(1) 2　(2) 不存在　(3) $e^2$

解：(1) $\lim\limits_{x\to 0}\dfrac{\sin\left(x^2\right)}{1-\cos x}=\lim\limits_{x\to 0}\left[\dfrac{\sin\left(x^2\right)}{x^2}\times\dfrac{x^2}{1-\cos x}\right]$

$=\lim\limits_{x\to 0}\left[\dfrac{\sin\left(x^2\right)}{x^2}\times\dfrac{4\times\left(\frac{x}{2}\right)^2}{1-\left(1-2\sin^2\frac{x}{2}\right)}\right]$

$=\lim\limits_{x\to 0}\dfrac{\sin\left(x^2\right)}{x^2}\times\lim\limits_{x\to 0}\dfrac{4}{2}\times\lim\limits_{x\to 0}\dfrac{\left(\frac{x}{2}\right)^2}{\sin^2\frac{x}{2}}=1\times 2\times 1=2$

(2) $\lim\limits_{x\to 0^+}\dfrac{\left|\tan x\right|}{1-\sqrt{1+2x}}=\lim\limits_{x\to 0^+}\left(\dfrac{\tan x}{x}\times\dfrac{x}{1-\sqrt{1+2x}}\right)$

$=\lim\limits_{x\to 0^+}\left[\dfrac{\sin x}{x}\times\dfrac{1}{\cos x}\times\dfrac{x\left(1+\sqrt{1+2x}\right)}{-2x}\right]$

$=\lim\limits_{x\to 0^+}\dfrac{\sin x}{x}\times\lim\limits_{x\to 0^+}\dfrac{1}{\cos x}\times\lim\limits_{x\to 0^+}\dfrac{1+\sqrt{1+2x}}{-2}=1\times 1\times(-1)=-1$

$$\lim_{x\to0^-}\frac{|\tan x|}{1-\sqrt{1+2x}}=\lim_{x\to0^-}\left(\frac{-\tan x}{x}\times\frac{x}{1-\sqrt{1+2x}}\right)$$

$$=\lim_{x\to0^-}\left[\frac{-\sin x}{x}\times\frac{1}{\cos x}\times\frac{x\left(1+\sqrt{1+2x}\right)}{-2x}\right]$$

$$=\lim_{x\to0^-}\frac{-\sin x}{x}\times\lim_{x\to0^-}\frac{1}{\cos x}\times\lim_{x\to0^-}\frac{1+\sqrt{1+2x}}{-2}=(-1)\times1\times(-1)=1$$

故 $\lim\limits_{x\to0}\dfrac{|\tan x|}{1-\sqrt{1+2x}}$ 不存在

(3) $x-1<[x]\le x\xrightarrow{1+\frac{2}{x}>1}\left(1+\frac{2}{x}\right)^{x-1}<\left(1+\frac{2}{x}\right)^{[x]}\le\left(1+\frac{2}{x}\right)^{x}$

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x-1}=\lim_{x\to\infty}\left[\left(1+\frac{2}{x}\right)^{\frac{x}{2}}\right]^2\times\frac{1}{1+\frac{2}{x}}$$

$$=\left[\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{\frac{x}{2}}\right]^2\times\lim_{x\to\infty}\frac{1}{1+\frac{2}{x}}=e^2\times1=e^2$$

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x}=\lim_{x\to\infty}\left[\left(1+\frac{2}{x}\right)^{\frac{x}{2}}\right]^2=\left[\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{\frac{x}{2}}\right]^2=e^2$$

故 $\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^{[x]}=e^2$