

每日一進

修行僧人，都能貫徹「一日不作，一日不食」的自律
修業儒生，豈能推諉「一日不進，一日不食」的自課

【週六版：大一微積分】

1、Find the following limit.

$$(1) \lim_{x \rightarrow 0} \sqrt[3]{x} \sin \left(\sin \left(\frac{1}{|x|} \right) \right)$$

$$(2) \lim_{x \rightarrow \infty} \sqrt[3]{x} \sin \left(\sin \left(\frac{1}{|x|} \right) \right)$$

$$(3) \lim_{x \rightarrow -\infty} \left(\sqrt{x^2 + 2x - 1} + x \right)$$

答：(1) 0 (2) 0 (3) -1

解：(1) $-1 \leq \sin \left(\sin \left(\frac{1}{|x|} \right) \right) \leq 1 \Rightarrow -|\sqrt[3]{x}| \leq \sqrt[3]{x} \sin \left(\sin \left(\frac{1}{|x|} \right) \right) \leq |\sqrt[3]{x}|$

$$\lim_{x \rightarrow 0} -|\sqrt[3]{x}| = 0, \quad \lim_{x \rightarrow 0} |\sqrt[3]{x}| = 0$$

$$\text{故 } \lim_{x \rightarrow 0} \sqrt[3]{x} \sin \left(\sin \left(\frac{1}{|x|} \right) \right) = 0$$

$$\begin{aligned} (2) \lim_{x \rightarrow \infty} \sqrt[3]{x} \sin \left(\sin \left(\frac{1}{|x|} \right) \right) &= \lim_{x \rightarrow \infty} \frac{\sin \left(\sin \left(\frac{1}{|x|} \right) \right)}{\frac{1}{\sqrt[3]{x}}} \\ &= \lim_{\substack{x \rightarrow \infty \\ \frac{1}{x} \rightarrow 0 \\ \frac{1}{|x|} \rightarrow 0 \\ \sin \left(\frac{1}{|x|} \right) \rightarrow 0}} \frac{\sin \left(\sin \left(\frac{1}{|x|} \right) \right)}{\sin \left(\frac{1}{|x|} \right)} \times \frac{\sin \left(\frac{1}{|x|} \right)}{\frac{1}{|x|}} \times \frac{\frac{1}{|x|}}{\frac{1}{\sqrt[3]{x}}} = 1 \times 1 \times 0 = 0 \end{aligned}$$

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$$\begin{aligned}
 (3) \quad & \lim_{x \rightarrow -\infty} \left(\sqrt{x^2 + 2x - 1} + x \right) \xrightarrow{t = -x} \lim_{t \rightarrow \infty} \left(\sqrt{t^2 - 2t - 1} - t \right) \\
 & = \lim_{t \rightarrow \infty} \left(\sqrt{t^2 - 2t - 1} - t \right) \times \frac{\left(\sqrt{t^2 - 2t - 1} + t \right)}{\left(\sqrt{t^2 - 2t - 1} + t \right)} = \lim_{t \rightarrow \infty} \frac{-2t - 1}{\left(\sqrt{t^2 - 2t - 1} + t \right)} \\
 & = \lim_{t \rightarrow \infty} \frac{-2 - \frac{1}{t}}{\left(\sqrt{1 - 2 \times \frac{1}{t} - \frac{1}{t^2}} + 1 \right)} = \frac{-2 - 0}{\sqrt{1 - 0 - 0} + 1} = -1
 \end{aligned}$$

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