

# 每日一進

修行僧人，都能貫徹「一日不作，一日不食」的自律  
修業儒生，豈能推諉「一日不進，一日不食」的自課

## 【週六版：大一微積分】

1、Compute each of the following limits if it exists or explain why it doesn't exist.

$$(1) \lim_{x \rightarrow 0} \sin\left(\frac{1}{x^2}\right) \sin x$$

$$(2) \lim_{x \rightarrow 0} \frac{\tan x}{\sqrt{1 - \cos 3x}}$$

$$(3) \lim_{x \rightarrow 0} (\cos x)^{\frac{2}{x^2}}$$

答：(1) 0 (2) 發散 (3)  $\frac{1}{e}$

解：(1)  $-1 \leq \sin\left(\frac{1}{x^2}\right) \leq 1 \Rightarrow 0 \leq \left|\sin\left(\frac{1}{x^2}\right)\right| \leq 1 \Rightarrow 0 \leq \left|\sin\left(\frac{1}{x^2}\right) \sin x\right| \leq |\sin x|$

$$\lim_{x \rightarrow 0} 0 = 0, \quad \lim_{x \rightarrow 0} |\sin x| = 0,$$

$$\text{故 } \left| \lim_{x \rightarrow 0} \sin\left(\frac{1}{x^2}\right) \sin x \right| = 0 \Rightarrow \lim_{x \rightarrow 0} \sin\left(\frac{1}{x^2}\right) \sin x = 0$$

$$(2) \lim_{x \rightarrow 0} \frac{\tan x}{\sqrt{1 - \cos 3x}} = \lim_{x \rightarrow 0} \left[ \frac{\sin x}{1} \times \frac{1}{\cos x} \times \frac{1}{\sqrt{1 - (1 - 2\sin^2 \frac{3x}{2})}} \right]$$

$$= \lim_{x \rightarrow 0} \left[ \frac{\sin x}{x} \times \frac{1}{\cos x} \times \frac{1}{\sqrt{2}} \times \frac{\frac{3x}{2}}{\sin \frac{3x}{2}} \times \frac{x}{\left|\frac{3x}{2}\right|} \right]$$

$$\lim_{x \rightarrow 0^+} \left[ \frac{\sin x}{x} \times \frac{1}{\cos x} \times \frac{1}{\sqrt{2}} \times \frac{\frac{3x}{2}}{\sin \frac{3x}{2}} \times \frac{x}{\left|\frac{3x}{2}\right|} \right] = 1 \times 1 \times \frac{1}{\sqrt{2}} \times 1 \times \frac{2}{3} = \frac{\sqrt{2}}{3}$$

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$$\lim_{x \rightarrow 0^-} \left[ \frac{\sin x}{x} \times \frac{1}{\cos x} \times \frac{1}{\sqrt{2}} \times \left| \frac{\frac{3x}{2}}{\sin \frac{3x}{2}} \right| \times \frac{x}{\left| \frac{3x}{2} \right|} \right] = 1 \times 1 \times \frac{1}{\sqrt{2}} \times 1 \times \frac{-2}{3} = \frac{-\sqrt{2}}{3}$$

故  $\lim_{x \rightarrow 0} \frac{\tan x}{\sqrt{1 - \cos 3x}}$  不存在

$$(3) \lim_{x \rightarrow 0} (\cos x)^{\frac{2}{x^2}} = \lim_{x \rightarrow 0} e^{\ln \left[ (\cos x)^{\frac{2}{x^2}} \right]} = \lim_{x \rightarrow 0} e^{\frac{2}{x^2} \ln(\cos x)} = e^{\lim_{x \rightarrow 0} \left[ \frac{2}{x^2} \ln(\cos x) \right]}$$

$$\xrightarrow{\lim_{x \rightarrow 0} x^2 = 0, \lim_{x \rightarrow 0} \ln(\cos x) = 0} \text{ 羅比達法則} = e^{\lim_{x \rightarrow 0} \left( \frac{2}{2x} \times \frac{-\sin x}{\cos x} \right)}$$

$$= e^{\lim_{x \rightarrow 0} \left( -\frac{\sin x}{x} \times \frac{1}{\cos x} \right)} = e^{-1} = \frac{1}{e}$$

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神貫注 力以赴