每日一進

修行僧人，都能貫徹「一日不作，一日不食」的自律
修業儒生，豈能推諉「一日不進，一日不食」的自課

【週六版：大一微積分】

1. Use implicit differentiation to find the tangent line at $(2,2)$ of the graph of the function $2x^3-3y^2=4$. Also find the second derivative $\frac{d^2y}{dx^2}$ at $(2,2)$.

答：(1) $y-2=2(x-2)$, i.e. $y=2x-2$ (2) 0

解：$2x^3-3y^2=4\Rightarrow 6x^2-6y\times\frac{dy}{dx}=0\Rightarrow 12x-6\left(\frac{dy}{dx}\times\frac{dy}{dx}+y\times\frac{d^2y}{dx^2}\right)=0$

故在$(2,2)$處，$6\times2^2-6\times2\times\frac{dy}{dx}=0\Rightarrow$ 切線斜率 $m=\left.\frac{dy}{dx}\right|_{\substack{x=2\\y=2}}=2$

則切線為$y-2=2(x-2)$，亦即$y-2=2(x-2)$

故在$(2,2)$處，$12\times2-6\left(2\times2+2\times\frac{d^2y}{dx^2}\right)=0\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{\substack{x=2\\y=2}}=0$

2. Set $f(x)=\begin{cases}x^2\sin\frac{1}{x}, & x\neq0\\ 0, & x=0\end{cases}$

(a)Find $f'(0)$.

(b)When $x\neq0$, find $f'(x)=?$

(c)Do's $f''(0)$ exist? If it exists, please find its value. If not, give reason to support your argument.

答：(a)0 (b)$2x\sin\frac{1}{x}-\cos\frac{1}{x}$ (c)not exist.

解：$\lim_{x\to0^+}\frac{f(x)-f(0)}{x-0}=\lim_{x\to0^+}x\sin\frac{1}{x}=\lim_{x\to0^+}\frac{\sin\frac{1}{x}}{\frac{1}{x}}=0$

$$\lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0^-}x\sin\frac{1}{x}=\lim_{x\to 0^-}\frac{\sin\frac{1}{x}}{\frac{1}{x}}=0$$

$$f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=0$$

解：When $x\neq 0$，x^2 與 $\sin\frac{1}{x}$ 處處可微分

故 $f'(x)=\frac{d}{dx}\left(x^2\sin\frac{1}{x}\right)=2x\sin\frac{1}{x}+x^2\cos\frac{1}{x}\times\left(-x^{-2}\right)=2x\sin\frac{1}{x}-\cos\frac{1}{x}$

解：$f'(x)=2x\sin\frac{1}{x}-\cos\frac{1}{x}$ 在 $x=0$ 處不存在，亦即 $f'(x)$ 在 $x=0$ 處不連續

故 $f''(0)$ 不存在