

每日一進

就算：學科再艱深，內容再枯澀
總要：自信地堅持，倔強地進步

【週六版：大一微積分】

1. Let $f(x) = \frac{\cot \frac{\pi}{2}x}{\ln x}$, $0 < x < 1$. Find $\lim_{x \rightarrow 1^-} f(x)$ and $\lim_{x \rightarrow 1^-} f'(x)$.

答： $\lim_{x \rightarrow 1^-} f(x) = -\frac{\pi}{2}$ 、 $\lim_{x \rightarrow 1^-} f'(x) = -\frac{\pi}{4}$

解： $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \frac{\cot \frac{\pi}{2}x}{\ln x}$

羅比達法則 $\rightarrow = \lim_{x \rightarrow 1^-} \frac{\left(-\csc^2 \frac{\pi}{2}x\right) \times \frac{\pi}{2}}{\frac{1}{x}} = \frac{-1 \times \frac{\pi}{2}}{1} = -\frac{\pi}{2}$

$$\lim_{x \rightarrow 1^-} f'(x) = \lim_{x \rightarrow 1^-} \frac{\ln x \times \left(-\csc^2 \frac{\pi}{2}x\right) \times \frac{\pi}{2} - \frac{1}{x} \times \cot \frac{\pi}{2}x}{(\ln x)^2}$$

$$= \lim_{x \rightarrow 1^-} \frac{-x \times \ln x \times \frac{\pi}{2} - \sin \frac{\pi}{2}x \times \cos \frac{\pi}{2}x}{(\ln x)^2} \times \frac{1}{x \times \sin^2 \frac{\pi}{2}x} \cdot \lim_{x \rightarrow 1^-} \frac{1}{x \times \sin^2 \frac{\pi}{2}x} = 1$$

羅比達法則 $\rightarrow = \lim_{x \rightarrow 1^-} \frac{-\frac{\pi}{2}(\ln x + 1) - \left(\cos^2 \frac{\pi}{2}x - \sin^2 \frac{\pi}{2}x\right) \times \frac{\pi}{2}}{2 \ln x \times \frac{1}{x}}$

$$= \lim_{x \rightarrow 1^-} \frac{-\frac{\pi}{2} \left(\ln x + 2 \cos^2 \frac{\pi}{2}x \right)}{2 \ln x} \times x \cdot \lim_{x \rightarrow 1^-} x = 1$$

羅比達法則 $\rightarrow = \lim_{x \rightarrow 1^-} \frac{-\frac{\pi}{2} \left(\frac{1}{x} - 4 \cos \frac{\pi}{2}x \sin \frac{\pi}{2}x \times \frac{\pi}{2} \right)}{2 \frac{1}{x}}$

$$= \frac{-\frac{\pi}{2} \left(1 - 4 \times 0 \times 1 \times \frac{\pi}{2} \right)}{2 \times 1} = -\frac{\pi}{4}$$

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克斌